Quantitative description of the dynamics in resonant islands

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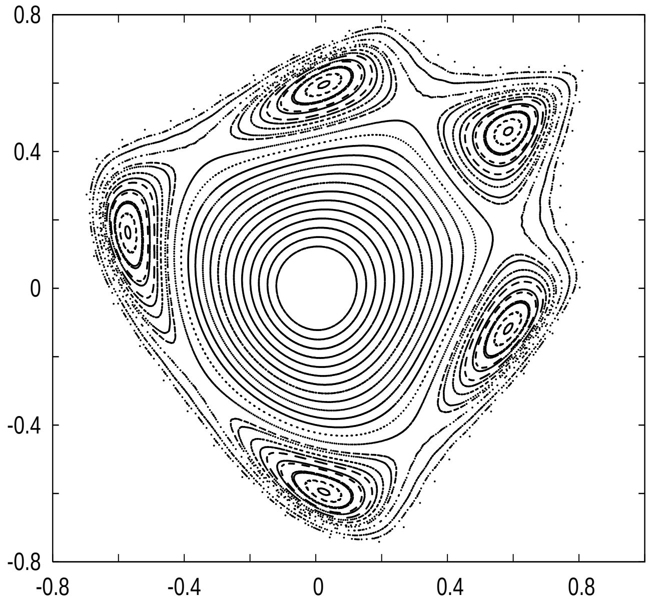
Goal

To describe the phase space of a one-parameter family of APMs (maybe a Poincaré map) around (but in a large domain, **global analysis**) of an elliptic fixed point E_0, that is, we look for **quantitative data** concerning the existence/destruction of stable motions, fixed/periodic points, local/global bifurcations, rotational invariant curves (r.i.c.) preventing transport, ...



“Pendulum-like” resonant islands emanate from E_0 as the parameter evolves. The separatrices of the penduli generically split.

Key ingredients

- KAM theory $\Rightarrow \exists$ of **r.i.c** around E_0.

  A given curve persists provided: 1) "good" **Diophantine** conditions of the rotation number, 2) size of the **perturbation** small enough, and 3) non-zero **"twist"** in the domain $\Rightarrow$ The most robust curves in the domain are those with noble rotation number $\rho = [a_0, ..., a_k, 1, 1, 1...]$ with smallest k.
- Splitting of separatrices: generically exponentially small with respect to a suitable "distance-to-integrable" parameter. In a **resonant island** there are two main splittings to take into account: the inner and outer ones. Generically they have a size of **different order of magnitude**. The splittings create **chaotic zones** in the phase space and the **interaction of resonances** can create chaotic zones of large size.

Resonant zones, inner and outer splittings in generic and low order resonances of area preserving maps.

Nonlinearity 22, 5:1191–1245, 2009.

Tools

We combine **analytical tools...**

$\longrightarrow$ normal form theory, interpolating Hamiltonian, averaging theory, KAM theory, complex singularities, asymptotics beyond all orders, simplified adapted models (standard map, separatrix map,...), ...

... with, rigorously implemented, **numerical techniques**

$\longrightarrow$ to implement theoretical schemes and check/improve/provide theoretical results (normal form algebraic manipulators, effective construction of the interpolating Hamiltonians, computation of invariant manifolds,...).

$\longrightarrow$ to perform massive simulations (size of the domain of stability, Lyapunov exponents, transport properties, rotation number (frequency map),...) both for discrete maps and flows.

Combining both approaches we get **quantitative** data on the system useful for **real applications**: where are the resonant islands located, size of them, size of the stability domain, size of the chaotic zones, transport velocity through different regions of the phase space,...